Project 12.19 on page 217 of the maple text

Area Between a Curve and Its Tangent Line

An example and instructions

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As an example, here is a solution in maple, for the first five steps as asked by the project, for function f(x) = -x^2. Please read the project 12.19 on page 217 of the maple text first. For your convenience, a copy of the project (page 217) is also posted with this lab.

```
> restart;
```

Step 1: Define a concave (up or down) function, find its derivative, and then define it as df(x). In this example, we use y = f(x) = -x^2 as suggested by the text.

```
> f:=x->-x^2;
> diff(f(x),x);
> df:=x->;
> df(x);
```

Step 2: Find the equation of tangent line to the curve at point x=p, which is a linear function in x with the parameter p.

```
> f_tang:=x->f(p)+df(p)*(x-p);
> f_tang(x);
```

Step 3: Compute the area between the curve and the tangent line over the interval [0,1]. We have Area = $\int_0^1 \left(f\_tang(x) - f(x)\right) \mathrm{d}x$, since the curve is below its tangent line in this case (why?). This area depends on where the line is tangent to the curve and therefore can be consider as a function of p, as you can see.

```
> f_area:=int(f_tang(x)-f(x)),x=0..1);
> f_area:=p->;
> f_area(p);
```

Step 4: Plot the area and find the value p_min where the area is the minimum. From the graph, we can see that the minimum occurred at about p=0.5. To find the critical point and confirm that this is a minimum by calculus, we use the first derivative test and then the second derivative test.

```
> plot(f_area(p),p=0...1);
> diff(f_area(p),p);
> df_area:=p->;
> p_min:=solve(df_area(p)=0,p);
> ddf_area:=diff(df_area(p),p);
> eval(ddf_area,p=p_min);
```

Step 5: Plot the curve and a few of its tangent lines, such as, at p=0.1, 02, 05, 08, and 0.9. Notice that we already plotted the area function in step 4.

```
> f1:=x->eval(f_tang(x),p=0.1);
> f2:=x->eval(f_tang(x),p=0.2);
> f5:=x->eval(f_tang(x),p=0.5);
> f8:=x->eval(f_tang(x),p=0.8);
> f9:=x->eval(f_tang(x),p=0.9);
> plot([f(x),f1(x),f2(x),f5(x),f8(x),f9(x)],x=0..1);
```

Here is what you need to do to complete the project:

1) In the above example of f(x)=-x^2, we see that p_min=0.5. Do you think that we will get a different p_min if we take a different f(x)? To find out, repeat the above steps for functions assigned by your TA and at least two more of your own (step 6). What do you think now? Make a conjecture about p_min based on your findings (step 7).

2) Repeat 1) but over a general interval [a, b] instead of [0, 1]. Make a more general conjecture (step 9).

3) Your finding is not a coincidence and there are some mathematical reasons behind it. Use calculus to prove our conjectures (**not by maple as asked in step 8**) and write up your report. It is not as hard as you may think. The examples you have worked out should give you hints on how to do the proof in general and your TA will help you, too.

You need to complete 1) and 2) in the first week and have your maple worksheet ready to turn in at the next lab meeting. You should also have worked on 3) and ready to ask questions (if any) by then. Your TA will decide when the full report is due.